Review Article

International Journal of Fundamental Research

Volume 1 Issue 1 (June - 2026) ISSN – xxxx-xxxx

Fuzzy Mathematics in Economics

Dr. Arun Kumar Mishra [a]*

[a] *Department of Mathematics, P.U.P. College, Motihari - 845401 (B.R.A. Bihar University, Muzaffarpur)*

ARTICLE INFO

Keywords:
Fuzzy Mathematics
Fuzzy Sets
Fuzzy Welfare Measure
Mathematical Economics

ABSTRACT

Fuzzy Mathematics is an expansion of fuzzy sets. An American Professor of Computer Science, Lofti Zadeh introduced the concept of fuzzy sets, in 1965. The capability of fuzzy sets to express gradual transitions from membership to non-membership and vice-versa has a wide utility. The major thrust of Economics is to describe the behaviours of individual units like consumers, householders, firms, government agencies and their interactions. Fuzzy Mathematics seems to be more suited in explaining the concepts of Economics than the classical results from our well-known Mathematics. In this paper, fuzzy welfare measure is discussed with a proper explanation.

1. INTRODUCTION

Many researchers use current money income as a proxy for a measure of economic welfare. Traditional money-revenue measure is obviously inadequate. Individuals at the same income level may vary substantially in their ability to consume goods and services. For population groups where earned income is a principal source of wellbeing or where human capital is more important than physical assets held, more attention should be devoted to the specification of the earning component. Income of a person is seemed to be exact. But the welfare that a person gets from the income is non- exact. The comparisons of inter personal welfare are made mainly by the methods of algebra. But in many real definite applications there are many difficulties due to rigidity of methods. So, it seems appropriate to model the concept of welfare in terms of fuzzy mathematical tool rather than in terms of classical Mathematics. According to Barr, "Full Income of a person consists of the flow of services from all individual wealth". That is.

$$Y_F = Y_M + Y_N \qquad (1.1)$$

where Y_F = Full Income, Y_M = Money Income and Y_N = Non-money Income. To combine aspects, related to human-wellbeing with income, 'Human Development Index (HDI) was introduced by United Nation's Development Programme in 1990. This index measures welfare of industrial countries. They identified the indications; the mortality rate, the life expectancy and the education attainment of members of the country so, we have,

$$\text{HDI} = 1 - \frac{1}{3}\sum_{i=1}^{3}\left\{\frac{\max_j X_{ij} - X_{ij}}{\max_j X_{ij} - \min_j X_{ij}}\right\} \qquad (1.2)$$

$j = 1, 2, \cdots, n$; X_{ij} = the value of the ith indicator for the individual j; $\max_j X_{ij}$ = the maximum value of the ith indicator in the country, and $\min_j X_{ij}$ = the minimum value of the ith indicator in the country.

Kakwani has introduced a class of 'Achievement Indices' to represent the actual levels of standard of living to measure the wellbeing of a country, over a period of time. This is formulated for a country at a point of time by

$$A_j = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{X_{ij} - X_{i\min}}{X_{i\max} - X_{i\min}}\right\} \qquad (1.3)$$

where, X_{ij} = the value of the ith indicator, $X_{i\min}$ = minimum value of the ith indicators, $X_{i\max}$ = maximum value of the ith indicator and n = the number of indicators.

* Corresponding author.
E-mail address: mishra1959arun@gmail.com (Dr. Arun Kumar Mishra)

View Online: https://pub.aietr.ac.in/IJFR/view-article?pii=S0000000026616562
Received 02 Feb 2026; Revised 04 Mar 2026; Accepted 09 Apr 2026; Available online 12 Apr 2026
Published 15 June 2026.

© 2026 Asha Institute of Educational Training & Research, all rights are reserved.

2. **Fuzzy Welfare Function**: Welfare is a fuzzy aspect rather than crisp. We denote 'Fuzzy Welfare Function by FWF & suppose that $S = \{1, 2, \cdots, n\}$ is a finite set, which represent a society of n individuals. Let there be an n vector, $y = (y_1, y_2, \cdots, y_n)$ introduced the income configuration of the individuals. Let I be the person in the society with the largest income and y_I be the income of the person. Let us take y_i/y_I and denote it by $\tilde{y}$.

$$\tilde{y} = \frac{y_i}{y_I} \tag{2.1}$$

The mapping $w_y: S \times S \to [0, 1]$ defined by

$$w_{ij} = \max\{\tilde{y}_i - \tilde{y}_j, 0\}; i, j \in S \tag{2.2}$$

is called the **Fuzzy Welfare Function** as the exact income and $\tilde{y}_i$ is the fuzzy income of the individual i. Using the notion of fuzzy income of an individual, a Fuzzy Welfare Function is defined, which is used to develop the fuzzy measure of welfare to compare the welfare of two individuals in a more suitable way. The function w_y assigns to every pair $(i, j) \in S \times S$ a real number between 0 and 1. If $w_y(i, j) > 0$, i is more well off than j. If $w_y(i, j) = 0$, i is not more well off than j.

In 1982, Amartya Sen said, "There are of course good resources to think that sometimes a richer person may have lower welfare than a poorer person, e.g., if he is cripple." This statement of welfare can be explained by 'Fuzzy Welfare Index' on the basis of 'Fuzzy Welfare Function' (FWF). For, let u be any parameter of the welfare of an individual in a society. Let α be a real number, $0 \leq \alpha < 1$ and let u_i be another real number, $0 \leq u_i < 1$. Take a function, $u: S \to [0,1]$ by $u(i) = u_i$; the 'Fuzzy welfare index' of the person I in the society S is defined as the real number,

$$w_i = (1 - \alpha)\tilde{y}_i + \alpha u_i \tag{2.3}$$

The value of w_i lies between 0 and 1. This equation defines the fuzzy convexity. So, we can modify FWF by

$$W_y = \max\{w_i - w_j, 0\} \tag{2.4}$$

This equation needs one more parameter u along with the income. More parameters can be included for finding the index.

3. **Conclusion:** Fuzzy Mathematics can explain many of the principles of Economics in its own fashion. In addition to the above discussed unit, Fuzzy poverty Index can also be used to explain the poverty of nation or of individual poverty Index, P was defined by Sen as, $P = H[I + (1 - I)G]$; under H for the head count ratio, I for the income gap ratio and G for the Gini co-efficient of the income distribution of the poor. This definition can be also put as the scalar fuzzy convexity. In this way every structure of Economics has a wide application of fuzzy mathematics.

4. **References:**

[1] Howie, J.M.: 'An Introduction to semigroup Theory': Academic Press, New York, 1976.

[2] Kakwani, N.K., 'On a Class of Poverty Measures' Ecomometrica, 48(1980).

[3] Klir, G.J., Yuan, B., 'Fuzzy Sets and Fuzzy Logic'. Prentice Hall, New Delhi, 2001.

[4] Ovchinnikov, S.; 'Transitive Fuzzy Ordering of Fuzzy Numbers', Fuzzy Sets and Systems, 30, (1989).